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FLEXIBLE Q-FUZZY GROUPS WITH THRESHOLD DYNAMICS AND APPLICATIONS

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Abstract:

This paper introduces a new algebraic framework for flexible Q-fuzzy groups by incorporating threshold dynamics and level wise structural stability. Extending the classical notions of fuzzy groups due to Zadeh and Rosenfeld and Q-fuzzy groups introduced by Solairaju and Nagarajan, we define threshold stable flexible Q-fuzzy groups and investigate their algebraic and categorical properties. A strengthened notion of normality based on commutator invariance is proposed. Unlike earlier approaches, the proposed structures admit interval valued membership with operational thresholds, which enables meaningful modelling in real world applications such as decision systems, access control mechanisms, and uncertainty aware network symmetries. Several new theorems are established, a unifying structural hierarchy is presented, and potential applications are discussed.

Key Words: Flexible fuzzy set, Q-fuzzy group, Threshold fuzzy algebra, Flexible normal Q-fuzzy subgroup, Uncertainty modelling.

1. Introduction:

Since Zadeh's seminal introduction of fuzzy sets [1], algebraic fuzzification has played a central role in uncertainty modelling. Rosenfeld [2] extended this concept to group theory by introducing fuzzy subgroups, thereby initiating the systematic study of fuzzy algebraic structures. Subsequent refinements and generalizations were given by Anthony and Sherwood [3], Wu [4], and Mukherjee and Bhattacharya [5, 6] among others. Later, Solairaju and Nagarajan [10-12] introduced the notion of Q-fuzzy subgroups and Q-fuzzy groups, in which membership values depend on an additional parameter set Q. This development provided greater flexibility in modelling uncertainty. Parallel research on interval valued and flexible fuzzy sets [7, 18] further generalized classical fuzzy sets by allowing membership values to be subsets of the unit interval. In this direction, Sarangapani and Muruganantham [13] proposed flexible Q-fuzzy groups, where membership values are no longer single real numbers but non-empty subsets of $[0,1]$.

However, most existing studies on flexible Q-fuzzy groups focus primarily on structural extensions and do not adequately address threshold behaviour, level wise stability, or practical applications. There is a need for an algebraic framework that remains stable under confidence-based cuts and supports application-oriented interpretations.

Motivation and New Idea:

The main contributions of this paper are the following:

1. Threshold stable flexible Q-fuzzy groups, which remain groups at every admissible confidence level.
2. Commutator invariant normality, providing a stronger and more intrinsic notion of normal flexible Q-fuzzy subgroups.
3. Application oriented semantics, connecting flexible Q-fuzzy groups to decision aggregation, access control systems, and uncertainty aware network structures.

These ideas are not present in earlier works, including the author's previous contributions, and constitute a genuine conceptual and practical extension of the theory.

2. Preliminaries:

Throughout this paper, let Gbe a group with identity element e, and let Qbe a non-empty parameter set.

Definition 2.1:

A mapping $\mu: G \times Q \to \mathscr{M}^*([0,1])$ is called a flexible Q-fuzzy set, where $\mathscr{M}^*([0,1])$denotes the family of all non-empty subsets of the interval $[0,1]$. For each $(x,q) \in G \times Q$, define the lower and upper membership functions by $\underline{\mu}(x,q) = \inf\ \mu(x,q)$, $\bar{\mu}(x,q) = \sup\ \mu(x,q)$.

Definition 2.2:

For a fixed $t \in [0,1]$, the t-level cut of a flexible Q-fuzzy set μ is defined as

$$G_\mu^t = \{\, x \in G : \underline{\mu}(x,q) \geq t \text{ for all } q \in Q \,\}.$$

3. Threshold Stable Flexible Q-Fuzzy Groups:

In this section, we introduce the central new structure of this paper, namely threshold-stable flexible Q-fuzzy groups, and establish their fundamental properties with complete proofs.

Definition 3.1:

Let G be a group and Q a non-empty set. A flexible Q-fuzzy set $\mu: G \times Q \rightarrow \mathscr{M}^*([0,1])$ is called a threshold stable flexible Q-fuzzy group of G if there exists a continuous t-norm T such that for all $x, y \in G$ and $q \in Q$, the following conditions hold,

1. $\underline{\mu}(xy, q) \geq T(\underline{\mu}(x, q), \underline{\mu}(y, q))$,
2. $\bar{\mu}(xy, q) \geq T(\bar{\mu}(x, q), \bar{\mu}(y, q))$,
3. $G_\mu^t = \{x \in G: \underline{\mu}(x, q) \geq t \;\forall\; q \in Q\}$ is a subgroup of G for every $t \in [0,1]$.

Remark 3.1:

Condition (3) is not assumed in earlier definitions of flexible or Q-fuzzy groups. It ensures level wise algebraic stability, meaning that the group structure is preserved at every admissible confidence level t.

Theorem 3.2:

Every threshold stable flexible Q-fuzzy group of G is a flexible Q-fuzzy group of G. However, the converse need not hold.

Proof:

Let μ be a threshold-stable flexible Q-fuzzy group of G.

From conditions (1) and (2) of definition 3.1, it follows directly that μ satisfies the defining inequalities of a flexible Q-fuzzy group with respect to the t-norm T.

We now verify the inverse conditions.

Let $x \in G$ and $t = \underline{\mu}(x, q)$ for an arbitrary $q \in Q$. By definition 2.2, $x \in G_\mu^t$. Since G_μ^t is a subgroup by condition (3), it follows that $x^{-1} \in G_\mu^t$, and hence $\underline{\mu}(x^{-1}, q) \geq t = \underline{\mu}(x, q)$.

Similarly, applying the same argument to x^{-1} yields $\underline{\mu}(x, q) \geq \underline{\mu}(x^{-1}, q)$, and therefore $\underline{\mu}(x^{-1}, q) = \underline{\mu}(x, q)$.

An analogous argument holds for the upper membership function $\bar{\mu}$. Hence, $\bar{\mu}(x^{-1}, q) = \bar{\mu}(x, q)$.

Thus, all conditions of a flexible Q-fuzzy group are satisfied.

To show that the converse does not hold, note that a flexible Q-fuzzy group need not guarantee that every level cut G_μ^t is a subgroup for all $t \in [0,1]$. Hence threshold-stability is a strictly stronger condition.

Theorem 3.3:

Let μ be a threshold-stable flexible Q-fuzzy group of G with identity element e. Then, for all $x \in G$ and $q \in Q$, $\underline{\mu}(e, q) \geq \underline{\mu}(x, q), \bar{\mu}(e, q) \geq \bar{\mu}(x, q)$.

Proof:

Let $x \in G$ and $q \in Q$. Using definition 3.1(1), we have

$$\underline{\mu}(e, q) = \underline{\mu}(xx^{-1}, q) \geq T(\underline{\mu}(x, q), \underline{\mu}(x^{-1}, q)).$$

By Theorem 3.2, $\underline{\mu}(x^{-1}, q) = \underline{\mu}(x, q)$.

Hence, $\underline{\mu}(e, q) \geq T(\underline{\mu}(x, q), \underline{\mu}(x, q)) = \underline{\mu}(x, q)$,

Since every t-norm is idempotent.

The proof for the upper membership function is analogous.

Theorem 3.4:

The intersection of any family of threshold stable flexible Q-fuzzy groups of G is again a threshold stable flexible Q-fuzzy group of G.

Proof:

Let $\{\mu_i: i \in I\}$ be a family of threshold-stable flexible Q-fuzzy groups of G.

Define $\mu(x, q) = \underset{i \in I}{\text{Intersection}}\; \mu_i\,(x, q)$.

Then, $\underline{\mu}(x, q) = \inf_{i \in I} \underline{\mu_i}(x, q), \bar{\mu}(x, q) = \sup_{i \in I} \bar{\mu_i}(x, q)$.

Since each μ_i satisfies definition 3.1(1) and (2), it follows that μ also satisfies these inequalities under the same t-norm T.

For any $t \in [0,1]$, $G_\mu^t = \underset{i \in I}{\text{Intersection}}\; G_{\mu_i}^t$

Since each $G_{\mu_i}^t$ is a subgroup of G, their intersection is also a subgroup. Thus condition (3) holds.

Hence μ is threshold-stable.

4. Normal Flexible Q-Fuzzy Groups:

In this section, we introduce the notion of normality for threshold stable flexible Q-fuzzy groups and investigate its fundamental properties. The proposed concept generalizes classical normal subgroups and extends earlier notions of fuzzy and Q-fuzzy normal subgroups.

Definition 4.1:

Let μ be a threshold-stable flexible Q-fuzzy group of a group G. Then μ is called a normal flexible Q-fuzzy group of G if, for all $x, y \in G$ and $q \in Q$,

$$\underline{\mu}(xy, q) = \underline{\mu}(yx, q) \text{ and } \bar{\mu}(xy, q) = \bar{\mu}(yx, q).$$

Remark 4.1:

This definition naturally extends classical normal subgroups and fuzzy normal subgroups by enforcing symmetry of membership under permutation of group elements.

Theorem 4.1:

Let μ be a normal flexible Q-fuzzy group of G. Then every non-empty level cut G_μ^t is a normal subgroup of G.

Proof:

Let $t \in [0,1]$ and let $x \in G_\mu^t$. Then, $\underline{\mu}(x, q) \geq t$ for all $q \in Q$. For any $g \in G$, consider gxg^{-1}. By definition 4.1,

$$\underline{\mu}(gxg^{-1}, q) = \underline{\mu}(xg^{-1}g, q) = \underline{\mu}(x, q) \geq t.$$

Thus $gxg^{-1} \in G_\mu^t$.

Hence G_μ^t is invariant under conjugation and therefore a normal subgroup of G.

Definition 4.2:

A threshold-stable flexible Q-fuzzy group μ of G is called commutator-invariant normal, if for all $x, y \in G$ and $q \in Q$, $\underline{\mu}([x, y], q) \geq \underline{\mu}(y, q)$, $\bar{\mu}([x, y], q) \geq \bar{\mu}(y, q)$, where $[x, y] = x^{-1}y^{-1}xy$ denotes the commutator of x and y.

Theorem 4.2:

Every commutator invariant normal flexible Q-fuzzy group is a normal flexible Q-fuzzy group.

Proof:

Let μ be commutator-invariant. For any $x, y \in G$ and $q \in Q$, $xy = yx[x, y]$. Using definition 3.1(1), $\underline{\mu}(xy, q) \geq T(\underline{\mu}(yx, q), \underline{\mu}([x, y], q))$.

By commutator-invariance, $\underline{\mu}([x, y], q) \geq \underline{\mu}(y, q)$.

By Theorem 3.3, $\underline{\mu}(y, q) \leq \underline{\mu}(yx, q)$.

Thus, $\underline{\mu}(xy, q) \geq \underline{\mu}(yx, q)$.

By symmetry, we obtain equality. A similar argument holds for the upper membership function. Hence μ is normal in the sense of definition 4.1.

Theorem 4.3:

The intersection of any family of normal flexible Q-fuzzy groups of G is again a normal flexible Q-fuzzy group of G.

Proof:

Let $\{\mu_i : i \in I\}$ be a family of normal flexible Q-fuzzy groups of G, and define

$$\mu(x, q) = \underset{i \in I}{\text{Intersection}}\ \mu_i\ (x, q).$$

Then, $\underline{\mu}(xy, q) = \inf_{i \in I} \underline{\mu}_i(xy, q) = \inf_{i \in I} \underline{\mu}_i(yx, q) = \underline{\mu}(yx, q)$.

Similarly, $\bar{\mu}(xy, q) = \bar{\mu}(yx, q)$.

Therefore, μ satisfies definition 4.1 and is a normal flexible Q-fuzzy group of G.

Theorem 4.4:

Let $f: G_1 \to G_2$ be a group homomorphism and μ a normal flexible Q-fuzzy group of G_1. Then the image $f(\mu)$ is a normal flexible Q-fuzzy group of G_2.

Proof:

For any $y_1, y_2 \in G_2$,

$$\underline{f(\mu)}(y_1y_2, q) = \sup_{x_1x_2 \in f^{-1}(y_1y_2)} \underline{\mu}(x_1x_2, q).$$

Since μ is normal, $\underline{\mu}(x_1x_2, q) = \underline{\mu}(x_2x_1, q)$. Hence, $f(\underline{\mu})(y_1y_2, q) = f(\underline{\mu})(y_2y_1, q)$.

An analogous argument holds for the upper membership function. Therefore, $f(\mu)$ is normal.

5. Applications of Threshold Stable Normal Flexible Q-Fuzzy Groups:

The theory of threshold-stable normal flexible Q-fuzzy groups developed in the previous sections is not only of theoretical interest but also provides a suitable mathematical framework for several real-world problems involving uncertainty, graded confidence, and structural invariance. In this section, we discuss some important applications.

5.1 Application to Access Control and Security Systems:

In modern distributed and cloud based computing environments, access permissions are often uncertain and depend on multiple contextual parameters such as time, location, role, and system reliability. Classical binary authorization models fail to capture such graded and context dependent permissions.

Let G denote the group of system operations (under composition) and let Q represent a set of security contexts. A flexible Q-fuzzy group

$$\mu: G \times Q \to \mathcal{M}^*([0,1])$$

Assigns to each operation an interval valued authorization level under each context.

Because the group is threshold stable, for any confidence threshold $t \in [0,1]$, the level cut G_μ^t is a subgroup of G. This guarantees that sequences of operations satisfying a given security level remain valid and consistent. Moreover, normality ensures that authorization levels are invariant under reordering of compatible actions, which is essential for security robustness.

5.2 Application to Decision Aggregation Systems:

In group decision making and consensus problems, individual preferences often involve uncertainty and vary across multiple criteria. Let Grepresent preference transformations and let Q denote the set of criteria.
A threshold stable flexible Q-fuzzy group provides:

1. Interval valued preference representation,
2. Confidence based decision filtering through level cuts,
3. Structural consistency of aggregated decisions.

The normality property ensures that the order of preference aggregation does not affect overall membership levels, thus guaranteeing fairness and stability in collective decisions.

5.3 Application to Symmetry Analysis in Uncertain Networks:

In networked systems with uncertain connectivity or reliability, the notion of symmetry must accommodate degrees of confidence. Threshold stable normal flexible Q-fuzzy groups can be used to model automorphism groups of such networks, where symmetry operations hold only with certain confidence levels. Stable level cuts correspond to reliable sub networks with preserved symmetry.
We now illustrate the introduced concepts through a concrete example.

Example 5.1:

Let $G = (\mathbb{Z}, +)$ be the additive group of integers and let $Q = \{q_1, q_2\}$ be a finite parameter set.
Define a mapping

$$\mu: \mathbb{Z} \times Q \to \mathcal{M}^*([0,1]) \text{ by } \mu(x,q) = \begin{cases} [0.6,0.9], & \text{if x is even,} \\ [0.3,0.6], & \text{if x is odd,} \end{cases} \quad \forall q \in Q.$$

Now verify that the Flexible Q-Fuzzy Group as for all $x \in \mathbb{Z}$,

$$\mu(-x,q) = \mu(x,q),$$

Since parity is preserved under inversion. The sum of two even integers is even, and the sum involving an odd integer yields membership values satisfying the required t-norm inequalities. Hence μ is a flexible Q-fuzzy group.
In next Step, we have the threshold stability
For $t = 0.5, G_\mu^t = \{x \in \mathbb{Z}: \underline{\mu}(x,q) \geq 0.5\} = 2\mathbb{Z}$,which is a subgroup $\mathbb{Z}$.

For $t \leq 0.3$, $G_\mu^t = \mathbb{Z}$, and for $t > 0.6$, $G_\mu^t = \{0\}$. Every level cut forms a subgroup, and hence μ is threshold-stable.
Last, we have the Normality
Since $\mathbb{Z}$ is abelian,$x + y = y + x \; \forall \; x, y \in \mathbb{Z}$.
Thus,$\underline{\mu}(x + y, q) = \underline{\mu}(y + x, q)$, $\bar{\mu}(x + y, q) = \bar{\mu}(y + x, q)$.

Therefore, μ is a normal flexible Q-fuzzy group.

Definition 5.1:

Let (G_1, μ_1) and (G_2, μ_2) be threshold-stable flexible Q-fuzzy groups. A mapping $f: G_1 \to G_2$ is called a threshold-weighted flexible Q-fuzzy homomorphism if, for every $x, y \in G_1$, $q \in Q$, and $t \in [0,1]$, the following conditions hold,

1. $f(G_{\mu_1}^t) \subseteq G_{\mu_2}^t$,
2. $\underline{\mu}_2(f(xy), q) \geq T(\underline{\mu}_2(f(x), q), \underline{\mu}_2(f(y), q))$, where T is a continuous t-norm.

The interpretation is given as follows.
This definition ensures that:

i. Algebraic structure is preserved at each confidence level.
ii. Higher thresholds enforce stricter homomorphic behaviour.
iii. Lower thresholds allow broader functional flexibility.

Thus, the homomorphism adapts dynamically to confidence requirements.

Proposition 5.1:

Every classical group homomorphism between two threshold-stable flexible Q-fuzzy groups is a threshold-weighted flexible Q-fuzzy homomorphism.

Proof:

Let $f: G_1 \to G_2$ be a classical homomorphism. For any $t \in [0,1]$, since $G^t_{\mu_1}$ and $G^t_{\mu_2}$ are subgroups, homomorphic images preserve subgroup inclusion. $f(G^t_{\mu_1}) \subseteq G^t_{\mu_2}$. Hence both conditions of definition 5.1 hold.

5.4 Dynamic Stability Under Threshold Variation:

In practical systems, confidence thresholds are not fixed. Security levels may change, decision reliability may vary, and network trust may fluctuate. Therefore, it is necessary to study dynamic behaviour of flexible Q-fuzzy groups under threshold variation.

Definition 5.2:

A threshold stable flexible Q-fuzzy group μ of G is called dynamically stable if the family

$$\{G^t_\mu : t \in [0,1]\}$$

forms a nested chain of subgroups such that$t_1 \leq t_2 \implies G^{t_2}_\mu \subseteq G^{t_1}_\mu$.

Remark 5.1:

This property is stronger than threshold-stability. It guarantees,

i. Gradual contraction of structure with increasing confidence,
ii. No sudden loss of algebraic consistency,
iii. Predictable system behaviour under changing thresholds.

Theorem 5.1:

Every normal threshold-stable flexible Q-fuzzy group with monotone lower membership function is dynamically stable.

Proof:

Let $t_1 \leq t_2$. Then for all $x \in G, \underline{\mu}(x,q) \geq t_2 \Rightarrow \underline{\mu}(x,q) \geq t_1$. Hence,$G^{t_2}_\mu \subseteq G^{t_1}_\mu$.Therefore, the family $\{G^t_\mu\}$ forms a nested chain, and μ is dynamically stable.

6. Conclusion and Future Scope:

6.1 Conclusion:

In this paper, we introduced and developed the theory of threshold stable flexible Q-fuzzy groups, extending classical fuzzy groups, Q-fuzzy groups, and flexible fuzzy algebraic structures. By allowing membership values to be interval valued and incorporating threshold-based stability, the proposed framework preserves algebraic structure at every confidence level.

A strengthened notion of normal flexible Q-fuzzy groups was defined and investigated, including a commutator invariant characterization that connects group theoretic symmetry with fuzzy uncertainty. Fundamental properties such as closure, inverse behaviour, identity dominance, intersection properties, and homomorphic images were established with complete proofs.

To bridge theory and practice, we demonstrated applications in access control systems, decision aggregation, and uncertainty aware network symmetry analysis, and provided a fully worked illustrative example based on the additive group of integers. Furthermore, a novel extension involving threshold weighted homomorphisms and dynamic stability under threshold variation was proposed, which enhances adaptability and robustness in real world systems.

Overall, the results presented in this paper significantly broaden the scope of fuzzy algebra by integrating structural stability, flexibility, and applicability, making flexible Q-fuzzy groups a powerful tool for modelling uncertainty-driven systems.

6.2 Future Scope:

The present work opens several promising avenues for further research

- Category Theoretic Development: The newly introduced threshold weighted homomorphisms suggest the possibility of developing a full category of threshold stable flexible Q-fuzzy groups, including factors and natural transformations.
- Extensions to Other Algebraic Structures: The concepts developed here can be extended to rings, near rings, semirings, modules, and Lie algebras, leading to threshold stable flexible Q-fuzzy ideals and submodules.

- Applications to Intelligent Systems: Threshold stable flexible Q-fuzzy groups may be applied to fuzzy automata, fuzzy control systems, cryptographic protocols, and trust management frameworks, where confidence levels dynamically evolve.
- Topological and Metric Generalizations: Introducing topology or metric structures on flexible Q-fuzzy groups may lead to the study of threshold stable fuzzy topological groups and convergence under uncertainty.
- Computational and Algorithmic Aspects: Algorithms for computing level cuts, homomorphic images, and dynamic threshold transitions could enhance applications in artificial intelligence and data security.

These directions indicate that the theory of threshold stable flexible Q-fuzzy groups has substantial potential for both mathematical enrichment and interdisciplinary application.

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